

New Forms of Continuity (δ - e -Continuous and Contra δ - e -Continuous)

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Abstract

In this paper, we explore the aspects of new forms of continuity, by utilizing generalized open sets which play a significant role in general topology and real analysis. In this paper we introduced and investigated a relatively new notion of open sets; namely δ - e -open sets. Also, the comparison of this new notion with e -open sets have been done. Additionally, we apply the notion of δ - e -open sets in topological space to present and study a new classes of functions called δ - e -continuous and contra δ - e -continuous as a new generalization of contra continuity, these functions were compared with e -continuous and contra e -continuous.

Keywords: δ - e -open sets, δ - e -continuous and contra δ - e -continuous.

صيغ جديدة للاتصال

δ - e المتصلة و δ - e غير المتصلة

الملخص

في هذه الورقة، نحن ندرس مظاهر الصيغ الجديدة للاتصال، باستعمال عمم المجموعات المفتوحة التي تلعب دور هام في الطوبولوجيا العامة والتحليل الحقيقي. في هذه الورقة نحن قدمنا وتحرينا فكرة جديدة نسبياً من المجموعات المفتوحة؛ تسمى δ - e المتصلة. أيضاً، مقارنة هذه الفكرة الجديدة مع e - المجموعات المفتوحة المعمولة. إضافة إلى ذلك، نحن نطبق فكرة δ - e المتصلة في الفضاء الطوبولوجي لتقديم ودراسة أصناف جديدة من الدوال تسمى δ - e المتصلة و δ - e غير المتصلة. كتعميم جديد لغير الاتصال، هذه الدوال قورنت مع e - المتصلة و e - غير المتصلة. الكلمات المفتاحية: δ - e المتصلة، δ - e المتصلة، δ - e غير المتصلة.

Introduction

Generalized open sets are new and well-known and important notions in topology and its applications. Many topologists are focusing their research on these topics and this amounted to many important and useful results. In this respect, the variously modified form of continuity by utilizing generalized open sets which play a significant role in general topology and real analysis. Levine (1963) introduced the notion of semi-open sets. According to Cameron (2007) this notion was Levine's most important contribution to the topology field.

Njastad (1965) introduced and investigated a weak form of open sets called α -open sets (originally called α -sets) in topological space and since the advent of these notions, several research papers with interesting results in different aspects came to existence. A subset A of a topological space $(X; \tau)$ is called preopen if $A \subseteq \text{Int}(Cl(A))$. The term 'preopen' was used for the first time by Mashhour et al. (1982). Monsef et al. (1983) introduced the notions of β -open sets and β -continuity in topological space.

The notion of b -open sets or sp -open sets was introduced by Andrijević (1996) and Dontchev and Przemski (1996) respectively. This type of sets was discussed by Ekici and Caldas (2004) under the name of γ -open sets. The class of b -open sets generates the same topology as the class of preopen sets. Since the advent of these notions, several research paper with interesting results in different respects came to existence.

Raychaudhuri and Mukherjee (1993) introduced the notion of δ preopen sets and Park et al. (1997) introduced the notion of δ -semiopen sets.

Hdeib (1982), define the concept of W -closed subset of a space $(X; \tau)$ if contains all of its condensation points. In Al-zoubi and Al-nashef (2003) it

was shown that the collection of all W -open subsets forms a topology that is finer than the original topology on X and many properties of that space were also discussed it seems that the whole idea of Hdeib (1982) and Al-zoubi and Al-nashef (2003) came from the very well-known facts about the standard topology on the real line.

Ekici (2008) introduced the notions of e -open set where the notion of e -open sets generalize the notions of δ - semiopen sets and δ -preopen sets.

Continuity on topological space, is an important basic subject in the study of General Topology and several branches of mathematics.

Dontchev (1996) introduced the notions of contra-continuity in topological spaces. He defined a function $f: X \rightarrow Y$ is contra continuous if the preimage of every open set of Y is closed in X . A new weaker form of this class of function introduced and investigated by Ekici (2008) called contra e -continuous function.

The aim of this work is focused on some new classes of continuity and contra continuity via new type of open sets which is δ - e -open sets. The following figure is an enlargement of some previous well-known figures. Note that none of the implications is reversible.

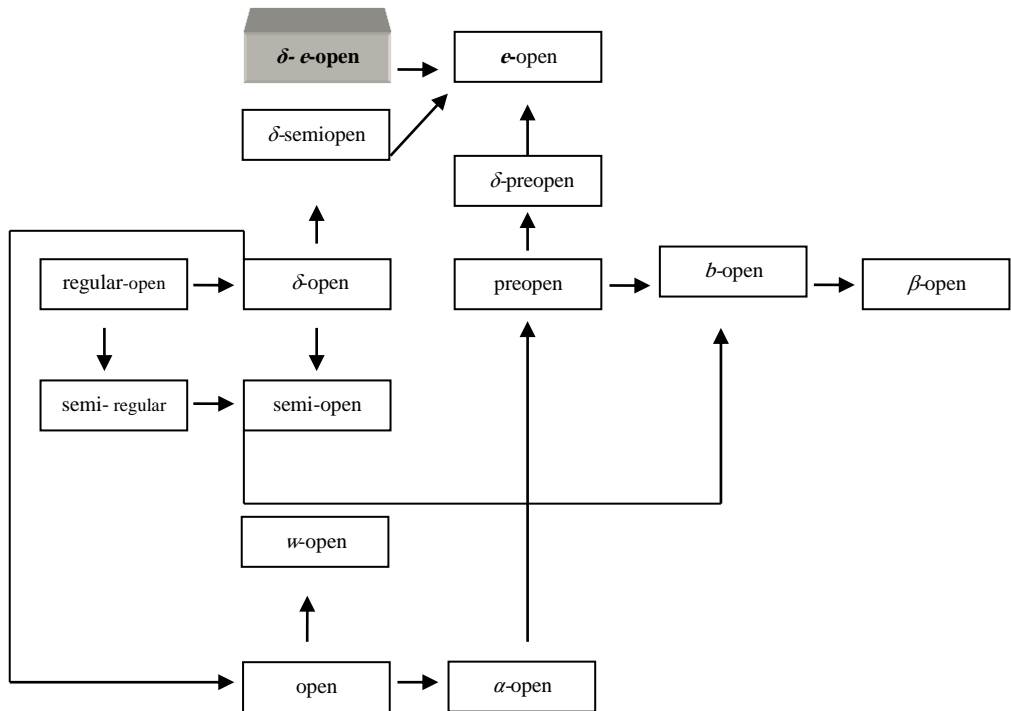


Figure 1 Relation among the sets in the topology τ

Concepts in Topology

Definition. (Adams and Franzosa 2008) Let X be a set. A topology τ on X is a collection of subsets of X , each called an open set, such that

1. \emptyset and X are open sets;
2. The intersection of finitely many open sets is an open set;
3. The union of any collection of open sets is an open set.

The set X together with a topology τ on X is called topological space, and it is denoted by (X, τ) .

Throughout the present paper, the space X and Y or $((X, \tau)$ and (Y, σ)) always mean topological spaces.

Definition. (Adams and Franzosa 2008) A subset A of a topological space X is closed if the set $X-A$ is open.

Definition. (Adams and Franzosa 2008) Let A be a subset of a topological space X . The interior of A , denoted $Int(A)$, is the union of all open sets contained in A . The closure of A , denoted $Cl(A)$, is the intersection of all closed sets containing A . For a subset A of a space X , the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively.

Definition. (Adams and Franzosa 2008) Let X and Y be topological spaces. A function $f: X \rightarrow Y$ is continuous if $f^{-1}(A)$ is open in X for every open set A in Y . We call this the open set definition of continuity.

Basic Concepts of e -Open Set

Let (X, τ) be a space and A is a subset of X . A subset A of a space X is said to be regular open (respectively regular closed) (Stone 1937) if $A = Int(Cl(A))$ (respectively $A = Cl(Int(A))$). Veličko (1968) introduced δ -interior of a subset A of X as the union of all regular open sets of X contained in A and is denoted by $\delta-Int(A)$. The subset A is called δ -open if $A = \delta-Int(A)$, i.e, a set is δ -open if it is the union of regular open sets. The complement of a δ -open set is called δ -closed. The δ -closure of a set A in a space (X, τ) is defined by: $\{x \in X : A \cap Int(Cl(B)) \neq \emptyset, B \in \tau \text{ and } x \in B\}$ and it is denoted by $\delta-Cl(A)$. The concepts of δ -open sets are a stronger notion of open set.

Definition. A subset A of a space (X, τ) is said to be:

1. (Levine 1963) semi-open if $A \subseteq Cl(Int(A))$ and semi-closed if $Int(Cl(A)) \subseteq A$;
2. (Njasted 1965) α -open if $A \subseteq Int(Cl(Int(A)))$ and α -closed if $Cl(Int(Cl(A))) \subseteq A$;

3. (Mashhour et al. 1982) preopen if $A \subseteq Int(Cl(A))$ and preclosed if $Cl(Int(A)) \subseteq A$;
4. (Monsef et al. 1983) β -open or (Andrijevič 1986) semi-preopen if $A \subseteq Cl(Int(Cl(A)))$ and β -closed if $Int(Cl(Int(A))) \subseteq A$;
5. (Raychaudhuri et ad. 1993) δ -preopen if $A \subset Int(\delta-Cl(A))$ and δ -preclosed if $Cl(\delta-Int(A)) \subset A$;
6. (Andrijevič 1996) b -open or (Dontchev and Przemski 1996) sp -open sets if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$;
7. (Park et al. 1997) δ -semiopen if $A \subset Cl(\delta-Int(A))$ and δ -semiclosed if $Int(\delta-Cl(A)) \subset A$;
8. (Ekici 2008) e -open if $A \subset Cl(\delta-Int(A)) \cup Int(\delta-Cl(A))$ and e -closed if $Cl(\delta-Int(A)) \cap Int(\delta-Cl(A)) \subset A$;

Let (X, τ) be a space and A a subset of X : A point x is a limit point of a set A if every open set containing x contains at least one point of A distinct from x . Particular kinds of limit point are W -accumulation point, for which every open set containing x must contain infinitely many points of A , and condensation points, for which every open set containing x must contain uncountably many points of A . A set A is said to be W -closed (Hdeib 1982) if it contains all its condensation points. The complement of an W -closed set is said to be W -open. It is well known that a subset W of a space (X, τ) is W -open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U - W$ is countable. We note that the collection of all α -open subset of X is a topology on X , called the δ -topology, which is finer than the original one. A

set $A \subseteq X$ is α -open if and only if A is semi-open and preopen set. Some authors use the term γ -open set for b-open set.

Definition. (Reilly 2000) Let A be a subset of a space X .

1. The α -closure of A , denoted by $\alpha\text{-Cl}(A)$, is the smallest α -closed set containing A . It is well-known that $\alpha\text{-Cl}(A) = A \cup \text{Cl}(\text{Int}(\text{Cl}(A)))$.
2. The α -interior of A , denoted by $\alpha\text{-Int}(A)$, is the largest α -open set contained in A .

Definition. (Ekici 2008) Let A be a subset of a space X .

1. The intersection of all e -closed sets containing A is called the e -closure of A and is denoted by $e\text{-Cl}(A)$.
2. The e -interior of A is defined by the union of all e -open sets contained in A and is denoted by $e\text{-Int}(A)$.

The following example show that none of the implications is reversible in figure 1.

Example. (Ekici 2008) Let $X = \{x, y, w, z\}$ and let $\tau = \{\emptyset, X, \{x\}, \{w\}, \{x, y\}, \{x, w\}, \{x, y, w\}, \{x, w, z\}\}$. Then:

1. the set $\{y, w\}$ is e -open but it is not β -open and so it is neither semiopen nor b -open.
2. the set $\{x, z\}$ is semiopen but it is not e -open so it is both b -open and δ -open.

Properties of e -Open Set

In order to prove our results and the benefit of the reader we recall some basic well known results.

Theorem. (Adams and Franzosa 2008) Let X be a space and A and B be subsets of X .

1. If U is an open set in X and $U \subset A$, then $U \subset Int(A)$.
2. If C is a closed set in X and $A \subset C$, then $Cl(A) \subset C$.
3. If $A \subset B$ then $Int(A) \subset Int(B)$.
4. If $A \subset B$ then $Cl(A) \subset Cl(B)$.
5. If A is open if and only if $A = Int(A)$.
6. A is closed if and only if $A = Cl(A)$.

Functions of e -Open Set

In this section, we recall some known notions, functions, and results which will be used in the work.

Definition. (Mashhour et al. 1983) A function $f : X \rightarrow Y$ is said to be α -continuous if $f^{-1}(V)$ is α -open in X for each open set V of Y .

Definition. A function $f : X \rightarrow Y$ is said to be:

1. (Dontchev 1996) contra-continuous if $f^{-1}(V)$ is closed in X for each open set of Y .

2. (Jafari and Noiri 2001) contra α -continuous if $f^{-1} (V)$ is α -closed in X for each open set of Y .

δ - e -Open Sets

Ekici (2008) defined and studied the notions of e -open sets. In this part we introduced relatively new notions called δ - e -open sets into the field of topology, in the same time we compare them with e -open sets. This comparison resulted in examples and theorems. Finally, new classes of functions called δ - e -continuous and contra δ - e -continuous functions by using the relatively new notions are introduced and studied. These have been compared with e -continuous and contra e -continuous, this is proved by examples and theorems.

Note that throughout this part: E stand for e .

δ - E -Open Sets

Definition. A subset A of a space X is said to be:

1. δ - e -open if $A \subset Cl(\delta-Int(A)) \cup \delta-Int(\delta-Cl(A))$.
2. δ - e -closed if $\delta-Cl(\delta-Int(A)) \cap Int(\delta-Cl(A)) \subset A$.

Based on Definition precedent we now illustrate the following examples:

Example. Let $X = \{x, y, w, z\}$ and let $\tau = \{\phi, X, \{x\}, \{w\}, \{x, y\}, \{x, w\}, \{x, y, w\}, \{x, w, z\}\}$.

Then:

1. the set $\{y, w\}$ is δ - e -open since $Cl(\delta-Int(\{y, w\})) \cup \delta-Int(\delta-Cl(\{y, w\})) = X$. Thus $\{y, w\}$ is δ - e -open.
2. the set $\{x, z\}$ is not δ - e -open because $Cl(\delta-Int(\{x, z\})) \cup \delta-Int(\delta-Cl(\{x, z\})) = \{x, y\}$. Thus $\{x, z\}$ is not δ - e -open.

Theorem. Every δ - e -open is e -open.

Proof. Let A is δ - e -open. Then we have $A \subset Cl(\delta-Int(A)) \cup \delta-Int(\delta-Cl(A))$. It is sufficient to show that $\delta-Int(U) \subset Int(U)$. Let $x \in \delta-Int(U)$. So there exists δ -open set; namely N such that $x \in N \subseteq U$. Since every δ -open is open. Then $x \in Int(U)$ and thus $\delta-Int(U) \subset Int(U)$. It follows that $\delta-Int(\delta-Cl(A)) \subset Int(\delta-Cl(A))$ and thus $Cl(\delta-Int(A)) \cup \delta-Int(\delta-Cl(A)) \subset Cl(\delta-Int(A)) \cup Int(\delta-Cl(A))$. We obtain $A \subset Cl(\delta-Int(A)) \cup Int(\delta-Cl(A))$. Hence A is e -open.

The following example shows that theorem precedent is not reversible.

Example. Let $X = \{x, y, z, w\}$ and let $\tau = \{ \phi, X, \{y\}, \{z\}, \{y, z\} \}$. Then the set $\{x\}$ is e -open since $Cl(\delta-Int(\{x\})) \cup Int(\delta-Cl(\{x\})) = \{x, y, w\}$. Thus $\{x\}$ is e -open but it is not δ - e -open because $Cl(\delta-Int(\{x\})) \cup \delta-Int(\delta-Cl(\{x\})) = \{y\}$. Thus $\{x\}$ is not δ - e -open.

Theorem. Every δ - e -closed is e -closed.

Proof. Let A is δ - e -closed. Then we have $\delta-Cl(\delta-Int(A)) \cap Int(\delta-Cl(A)) \subset A$. It is sufficient to show that $Cl(A) \subset \delta-Cl(A)$. Let $x \in Cl(A)$. For all U be open set we have $A \cap U \neq \emptyset$, but $U \subset Cl(U)$ and $Int(U) \subset Cl(Int(U))$. Since U is open and $U \subset Int(Cl(U))$. Then $A \cap U \subset A \cap Int(Cl(U))$. But $A \cap U \neq \emptyset$, then $A \cap Int(Cl(U)) \neq \emptyset$. Thus $x \in \delta-Cl(A)$. It follows that $Cl(\delta-Int(A)) \subset \delta-Cl(\delta-Int(A))$ and thus $Cl(\delta-Int(A)) \cap Int(\delta-Cl(A)) \subset \delta-Cl(\delta-Int(A)) \cap Int(\delta-Cl(A))$. We obtain $Cl(\delta-Int(A)) \cap Int(\delta-Cl(A)) \subset A$. Hence A is e -closed.

The converse implications do not hold as it is shown in the following example.

Example. Let $X = \{a, b, c, d\}$ and let $\tau = \{ \phi, X, \{a, d\}, \{c\}, \{a, c, d\} \}$. Then the set $\{a, c\}$ is e -closed since $Cl(\delta-Int(\{a, c\})) \cap Int(\delta-Cl(\{a, c\})) = \{b, c\}$.

Thus $\{a, c\}$ is e -closed but it is not δ - e -closed because $\delta\text{-Cl}(\delta\text{-Int}(\{a, c\})) \cap \text{Int}(\delta\text{-Cl}(\{a, c\})) = \{b, c\}$. Thus $\{a, c\}$ is not δ - e -closed.

Definition. Let A be a subset of a space X .

1. The intersection of all δ - e -closed sets containing A is called the δ - e -closure of A and is denoted by δ - e - $\text{Cl}(A)$.
2. The δ - e -interior of A is defined by the union of all δ - e -open sets contained in A and is denoted by δ - e - $\text{Int}(A)$.

Lemma. The following hold for a subset A of a space X : $X \setminus \delta$ - e - $\text{Cl}(A) = \delta$ - e - $\text{Int}(X \setminus A)$.

Proof. Let $x \in X \setminus \delta$ - e - $\text{Cl}(A)$. Then $x \notin \delta$ - e - $\text{Cl}(A)$. There exists U is δ - e -open such that $x \in U \cap A = \emptyset$. So $x \in U \subseteq X \setminus A$. Thus $x \in \delta$ - e - $\text{Int}(X \setminus A)$.

Theorem. Let X be a space, the following statements hold:

1. The union of any family of δ - e -open sets is an δ - e -open set.
2. The intersection of any family of δ - e -closed sets is an δ - e -closed set.

Proof. 1. Let $\{U_\alpha : \alpha \in \Gamma\}$ be any family of δ - e -open set.

Then we have $U_\alpha \subset \text{Cl}(\delta - \text{Int}(U_\alpha)) \cup \delta - \text{Int}(\delta - \text{Cl}(U_\alpha))$ for all $\alpha \in \Gamma$.

Now

$$\begin{aligned} \bigcup_{\alpha \in \Gamma} U_\alpha &\subset \bigcup_{\alpha \in \Gamma} (\text{Cl}(\delta - \text{Int}(U_\alpha)) \cup \delta - \text{Int}(\delta - \text{Cl}(U_\alpha))); \\ &= \bigcup_{\alpha \in \Gamma} (\text{Cl}(\delta - \text{Int}(U_\alpha)) \cup \delta - \text{Int}(\delta - \text{Cl}(U_\alpha))); \\ &= \text{Cl}(\bigcup_{\alpha \in \Gamma} \delta - \text{Int}(U_\alpha)) \cup \delta - \text{Int}(\bigcup_{\alpha \in \Gamma} \delta - \text{Cl}(U_\alpha)); \\ &= \text{Cl}(\delta - \text{Int}(\bigcup_{\alpha \in \Gamma} U_\alpha)) \cup \delta - \text{Int}(\delta - \text{Cl}(\bigcup_{\alpha \in \Gamma} U_\alpha)). \end{aligned}$$

Thus $\bigcup_{\alpha \in \Gamma} U_\alpha$ is δ - e -open.

2. Let $\{U_\alpha : \alpha \in \Gamma\}$ be any family of δ - e -closed. Then we have $\delta - \text{Cl}(\delta - \text{Int}(U_\alpha)) \cap \text{Int}(\delta - \text{Cl}(U_\alpha)) \subset U_\alpha$

for all $\alpha \in \Gamma$. Now

$\bigcap_{\alpha \in \Gamma} (\delta - Cl(\delta - Int(U_\alpha)) \cap Int(\delta - Cl(U_\alpha))) \supset \delta - Cl(\bigcap_{\alpha \in \Gamma} \delta - Int(U_\alpha))$
 $\cap Int(\bigcap_{\alpha \in \Gamma} \delta - Cl(U_\alpha)) \supset \delta - Cl(\delta - Int(\bigcap_{\alpha \in \Gamma} U_\alpha)) \cap Int(\delta - Cl(\bigcap_{\alpha \in \Gamma} U_\alpha)).$
 Then $\bigcap_{\alpha \in \Gamma} U_\alpha \supset (\delta - Cl(\delta - Int(\bigcap_{\alpha \in \Gamma} U_\alpha)) \cap Int(\delta - Cl(\bigcap_{\alpha \in \Gamma} U_\alpha))).$
 Thus $\bigcap_{\alpha \in \Gamma} U_\alpha$ is δ - e -closed.

δ -E-Continuous and Contra δ -E-Continuous

The notions of e -continuous and contra e -continuous were introduced and investigated by Ekici (2008). In this section, we apply the notion of δ - e -open sets in topological space to present and study a new class of functions called δ - e -continuous and contra δ - e -continuous functions.

δ - e -Continuous

Definition. (Ekici 2008) A function $f : X \rightarrow Y$ is said to be e -continuous if $f^{-1}(B)$ is e -open in X for every open set B of Y .

Definition. A function $f : X \rightarrow Y$ is said to be δ - e -continuous if $f^{-1}(A)$ is δ - e -open in X for every open set A of Y .

Theorem. Every δ - e -continuous functions is e -continuous functions.

Proof. Let $f : X \rightarrow Y$ be δ - e -continuous. Let V be open set in Y . Then $f^{-1}(V)$ is δ - e -open in X . By theorem (Every δ - e -open is e -open) we obtain $f^{-1}(V)$ is e -open in X . Hence f is e -continuous.

Example. Let $X = Y = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then the function $f : (X, \tau) \rightarrow (Y, \tau)$, defined as: $f(a) = b, f(b) = a, f(c) = a$ is δ - e -continuous since $f^{-1}(\{a\}) = \{b, c\}$ which is δ - e -open.

Example. Let $X = Y = \{x, y, w, z\}$ and $\tau = \{\emptyset, X, \{y\}, \{z\}, \{y, z\}\} = \delta$. Then the function $f: (X, \tau) \rightarrow (Y, \delta)$, defined as: $f(x) = y, f(y) = x, f(w) = z, f(z) = x$ is neither δ - e -continuous nor continuous because $f^{-1}(\{y\}) = \{x\}$ which is neither δ - e -open nor open.

Contra δ - e -Continuous

A function $f: X \rightarrow Y$ is said to be (Ekici 2008) contra e -continuous if $f^{-1}(B)$ is e -closed in X for every open set B of Y . In this section, we introduce a new type of continuity called contra δ - e -continuous.

Definition. A function $f: X \rightarrow Y$ is called contra δ - e -continuous if $f^{-1}(A)$ is δ - e -closed in X for every open set A of Y .

Theorem. Every contra δ - e -continuous functions is contra e -continuous functions.

Proof. Let $f: X \rightarrow Y$ be contra δ - e -continuous. Let V be open set in Y . Then $f^{-1}(V)$ is δ - e -closed in X . By theorem 3.2.5 we obtain $f^{-1}(V)$ is e -closed in X . Hence f is contra e -continuous.

Example. Let $X = Y = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Let $f: X \rightarrow Y$ be a function defined by $f(a) = b, f(b) = a, f(c) = a$. Then, f is contra δ - e -continuous since $f^{-1}(\{a\}) = \{b, c\}$ which is δ - e -closed.

Example. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, X, \{c\}, \{b\}, \{c, b\}\}$. Then the identity function $f: X \rightarrow Y$ is contra δ - e -continuous but it is not contra-continuous since $f^{-1}(\{b\}) = \{b\}$ which is δ - e -closed not closed.

Example. Let $X = Y = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Let $f: X \rightarrow X$ defined by $f(a) = c$, $f(b) = a$, $f(c) = c$, $f(d) = a$ is neither contra e -continuous nor contra δ - e -continuous because $f^{-1}(\{c\}) = \{a, c\}$ which is neither e -closed nor δ - e -closed.

Conclusion

In this paper we have proposed relatively new notions of open sets; namely δ - e -open set, this has been compared with e -open set using examples and theorems, as well as introducing new classes of functions called δ - e -continuous and contra δ - e -continuous. These functions were compared with e -continuous and contra e -continuous. The comparison resulted in examples and theorems.

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